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Atom cooling with an atom-optical diode on a ring

A. Ruschhaupt[1, *] and J. G. Muga[2, †]

[1] *Institut für Mathematische Physik, TU Braunschweig, Mendelssohnstrasse 3, 38106 Braunschweig, Germany*

[2] [b] *Departamento de Química-Física, Universidad del Pais Vasco 48080 Bilbao, Spain*

We propose a method to cool atoms on a ring by combining an atom diode –a laser valve for one-way atomic motion which induces robust internal state excitation– and a trap. We demonstrate numerically that the atom is efficiently slowed down at each diode crossing, and it is finally trapped when its velocity is below the trap threshold.

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There is currently much interest in controlling the motion of cold atoms for further (deeper) cooling, quantum information processing, atom laser generation, metrology, interferometry, and the investigation of fundamental physical phenomena. Cold atoms are relatively easy to produce and offer with respect to other particles many possibilities for coherent external manipulation with lasers, magnetic fields, or mechanical interactions. They may be trapped in artificial lattices or even individually, can be guided in effectively one-dimensional wires, or adopt interesting collective behavior in Bose-Einstein condensates; also, their mutual interactions can be changed, or suppressed. All this flexibility facilitates the translation of some of the concepts and applications of electronic circuits into the atom-optical realm to implement atom chips, atom circuits, or quite generally "atomtronics" [1]. In this context, efficient elementary circuit elements playing the role of diodes or transistors need to be developed. In particular, we have proposed and studied a laser device acting as a one-way barrier for atomic motion [2, 3, 4, 5, 6], and similar ideas, near experimental verification, have been considered by Raizen and coworkers for atom cooling [7, 8]. (These one-way models rely on atom-laser interactions in the independent atom regime, but there are also complementary proposals making use of interatomic interaction for "diodic" one-way transport [9].) In the following we propose to combine, within a ring, the diode and a trap, to achieve cooling and trapping with phase-space compression. Ring-shaped traps for cold atoms have been proposed or implemented for matter-wave interferometry and highly precise sensors [10], for studying the stability of persistent currents [11, 12], sound waves, solitons and vortices in Bose-Einstein condensates [13], collisions [14], for coherent acceleration [15, 16], production of highly directional output beams [17], or quantum computation [18]. (For further applications see [19].) The ring traps are implemented by magnetic waveguides [10, 17, 20], purely optical dipole forces [21], magnetoelectrostatic potentials [22], overlapping of magnetic and optical dipole traps [23], or misalignment of counterpropagating laser beam pairs in a magneto-optical trap [14].

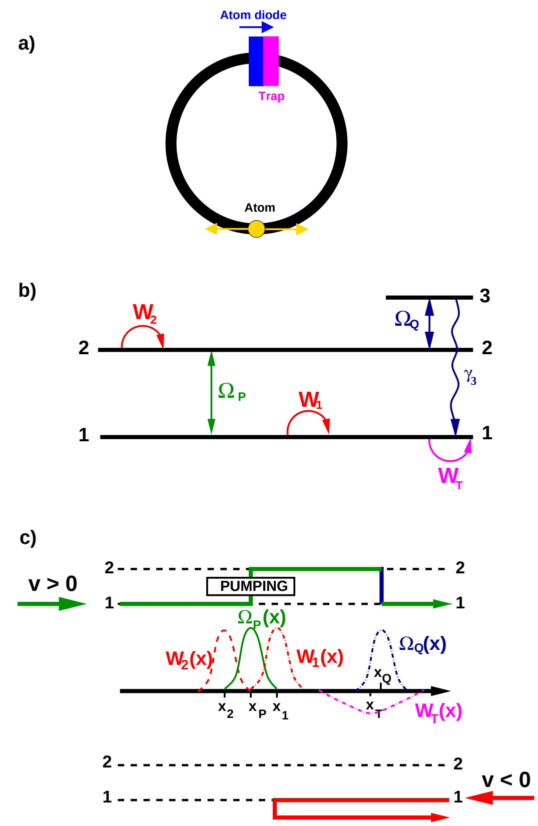

FIG. 1: (a) Setting of an atom diode in a ring, (b) schematic action of the different lasers on the atom levels for the two-level atom diode plus quenching, and (c) schematic spatial location of the different laser potentials and their effect on the moving atom. They are all taken as Gaussian functions: $\Omega_P(x) = \hat{\Omega}_P \Pi(x, x_P, \sigma)$, $W_\alpha(x) = \hat{W}_\alpha \, \Pi(x, x_\alpha, \sigma_\alpha)$, where $\alpha = 1, 2, T, Q$; $\sigma_{1,2} = \sigma$; and $\Pi(x, x_0, \tilde{\sigma}) = \exp\left(-\frac{(x-x_0)^2}{2\tilde{\sigma}^2}\right)$.

In this paper, we assume for simplicity tight lateral confinement so that motion along the ring is effectively one dimensional or, more exactly, circular with a length l. An atom diode followed by a trap are put on the ring (see Fig. 1a). The initial state of ground-state atoms will have some velocity and position width, but the anti-clockwise

moving atoms will be reflected by the diode -which can be crossed only in the direction of the arrow- so all atoms will eventually approach the diode clockwise. After each crossing of the diode, the atom, which is now excited, is forced to emit a spontaneous photon (quenched), and ends up at the bottom of the trap; the atom has to loose kinetic energy to escape from the trap, so it is slowed down at every crossing until being finally trapped when its velocity is below the trap threshold. The process is reminiscent of Sysiphus cooling [24], a difference being that both the transfer from ground to excited state in the diode and the quenched decay are highly controlled, robust and efficient processes.

Before looking at the quantum-mechanical description, we will examine a simple classical model to estimate the time scales and the cooling efficiency for different recoil velocities. In this classical toy model, the diode and the trap are reduced to a point at position x_D, and the initial particle positions and momenta are distributed according to Gaussian probability distributions corresponding to the initial quantum distributions $|\Psi_0(x)|^2$ resp. $|\Phi_0(k)|^2$ (see below). In each classical trajectory a random recoil kick is imparted at the diode clockwise passage, as in the quantum jump calculation done below. The trajectory is "trapped" (and eliminated from the ensemble) when the velocity becomes smaller than the threshold imposed by the trap depth; otherwise a fixed amount of kinetic energy corresponding to the well depth $\frac{1}{2}mv_T^2$ is subtracted and the motion continues. The results for the trapping probability are shown in Fig. 2a. Random recoil affects the result in two ways: higher recoil velocities accelerate a rapid initial trapping, but slightly increase the time necessary for cooling and trapping the complete ensemble. The number of diode crossings before the atom is trapped for an initial velocity $v > 0$ and no recoil is given by the smallest integer n fulfilling $v - nv_T < v_T$. The time until this particle has been trapped is given by the time to reach the diode the first crossing, $t_0 = -(x_0 - x_D)/v$ plus the total time for the n rounds, $t_n = l\sum_{j=1}^{n}(v - jv_T)^{-1}$. For the parameters of Fig. 2a and $v = v_0$ we get $n = 2$ and $t_0 + t_n \approx 41\,\mathrm{ms}$.

Now we switch to the quantum mechanical description. The scheme of the diode used here can be seen in Fig. 1b and c and it has been explained before [2, 3, 4, 6]. In brief, there are three, partially overlapping laser regions: two of them are state-selective mirrors blocking the excited (level 2) and ground (level 1) state on the left and right, respectively of a central pumping region on resonance with the atomic transition. If the atom is traveling from the right and the velocity is not too high, it is reflected by the state-selective mirror potential $W_1\hbar/2$. If the atom is traveling from the left in the ground state then it will be pumped to the second level adiabatically (so that the process is robust and independent of velocity in a broad velocity interval) and then pass the potential $W_1\hbar/2$. Note that this setting, see Fig. 1b, can be realized by a detuned STIRAP transfer [3] with just two overlapping lasers. To avoid backwards motion after the atom has crossed the diode we assume a third level which decays to the ground state with a decay rate γ_3 and we add a quenching laser coupling levels 2 and 3 with a Rabi frequency Ω_Q, see Fig. 1b. A novelty with respect to previous diode models is the addition of a ground state well overlapping partially with the quenching laser region. The effect of this well is twofold: it subtracts kinetic energy from the ground state atoms trying to escape from it, and it also traps eventually the cooled atoms. The corresponding Hamiltonian using $|1\rangle = (1,0,0)^T$, $|2\rangle = (0,1,0)^T$, and $|3\rangle = (0,0,1)^T$, where T means "transpose", may now be written as

$$H = \frac{\mathbf{p}_x^2}{2m} + \frac{\hbar}{2}\begin{pmatrix} W_1(x) + W_T(x) & \Omega_P(x) & 0 \\ \Omega_P(x) & W_2(x) & \Omega_Q(x) \\ 0 & \Omega_Q(x) & 0 \end{pmatrix},$$

where $\mathbf{p}_x$ is the momentum operator, and $\Omega_P(x)$ is the Rabi frequency for the resonant transition. All potentials are chosen as Gaussian functions according to the caption of Fig. 1. The "velocity depth" of the trap used is $v_T := \sqrt{\frac{\hbar}{m}\left|\hat{W}_T\right|} \approx 1.8\,\mathrm{cm/s}$, and it corresponds to the trap depth used in the classical simulation.

We examine the time evolution by means of a one-dimensional master equation,

$$\frac{\partial}{\partial t}\rho \;=\; -\frac{i}{\hbar}[H,\rho]_- - \frac{\gamma_3}{2}\{|3\rangle\langle 3|,\rho\}_+ + \gamma_3\int_{-1}^{1}du\,\frac{3}{8}(1+u^2)\,\exp\left(i\frac{mv_{rec}}{\hbar}u\mathbf{x}\right)|1\rangle\langle 3|\rho|3\rangle\langle 1|\exp\left(-i\frac{mv_{rec}}{\hbar}u\mathbf{x}\right), \quad (1)$$

where v_{rec} is the recoil velocity and $\mathbf{x}$ is the position operator. The initial condition is taken as a pure state $\rho(0) = |\Psi_0\rangle\langle\Psi_0|$, namely a Gaussian wave packet (see the caption of Fig. 2).

The master equation (1) is solved by using the quantum jump approach [27]. A basic step is to solve a time-dependent Schrödinger equation with an effective Hamiltonian $H_{eff} = H - i\frac{\hbar}{2}\gamma_3|3\rangle\langle 3|$. For large γ_3 (see [26]),

$$\langle 3|\Psi(t)\rangle \approx -i\frac{\Omega_Q(x)}{\gamma_3}\langle 2|\Psi(t)\rangle. \quad (2)$$

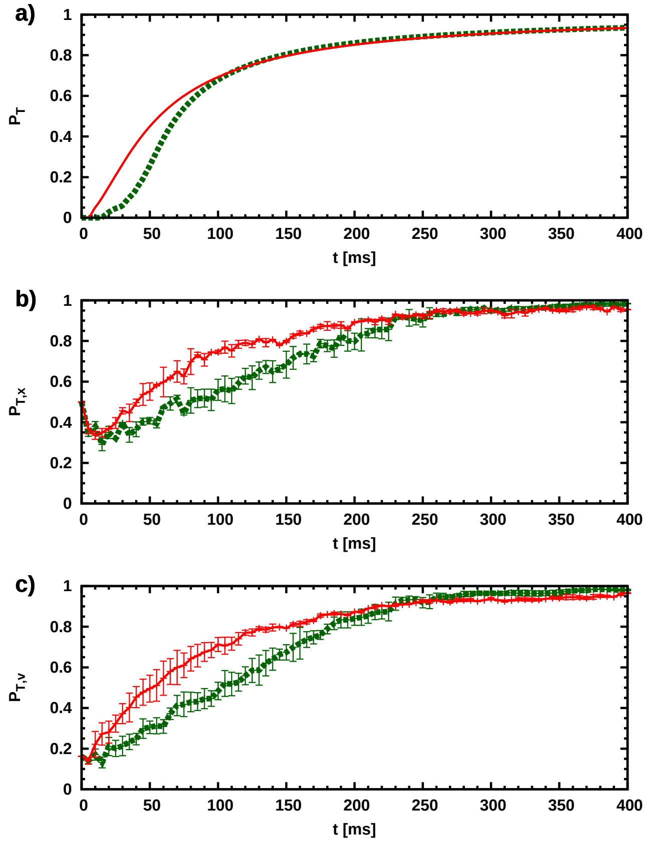

FIG. 2: (a) Trapping probability in the classical model; (b) trapping probability $P_{T,x}$ in the quantum model ($x_{min} = 10\,\mu$m and $x_{max} = 200\,\mu$m); and (c) trapping probability $P_{T,v}$ in the quantum model. Thick, green, dotted line: $v_{rec} = 0$ (in the quantum model: averaged over $N = 200$ trajectories, $\hat{\Omega}_P = 4\times10^4$/s, $\hat{W}_1 = \hat{W}_2 = 4\times10^6$/s); solid, red line: $v_{rec} = 3.5\,$cm/s (in the quantum model: averaged over $N = 190$ trajectories, $\hat{\Omega}_P = 1\times10^5$/s, $\hat{W}_1 = \hat{W}_2 = 1\times10^7$/s). Common parameters: $l = 400\,\mu$m ($-200\,\mu$m $\leq x < 200\,\mu$m), m mass of Neon, initial wave packet $\Psi_0(x) = \frac{1}{\sqrt{2\pi}}\int dk\,\Phi_0(k)e^{ikx}$ with $\Phi_0(k) = \frac{1}{\sqrt[4]{2\pi}\sqrt{\Delta k}}(1,0,0)^T \exp\Big[-\frac{(k-k_0)^2}{4\Delta k^2} - i(k-k_0)(x_0 - \frac{\hbar}{m}t_0k_0) - i\frac{\hbar}{m}t_0\frac{k^2}{2}\Big]$, $k_0 = \frac{m}{\hbar}v_0$, $\Delta k = \frac{m}{\hbar}\Delta v$, $x_0 = -200\,\mu$m, $v_0 = 5\,$cm/s, $\Delta v = 4\,$cm/s, $t_0 = 1\,$ms; other parameters in the classical model: $v_T = 1.8\,$cm/s, $x_D = 80\,\mu$m; other parameters in the quantum model: $\hat{W}_T = -10^5$/s, $\hat{W}_Q = 10^5$/s, $x_{W2} = -90\,\mu$m, $x_P = -40\,\mu$m, $x_{W1} = 10\,\mu$m, $x_T = 80\,\mu$m, $x_Q = 100\,\mu$m, $\sigma = 15\,\mu$m, $\sigma_T = 30\,\mu$m, $\sigma_Q = 10/\sqrt{2}\,\mu$m.

Therefore, the three-level Schrödinger equation can be approximated by a two-level one with the effective Hamiltonian

$$H_{approx} = \frac{\mathbf{p}_x^2}{2m} + \frac{\hbar}{2}\begin{pmatrix} W_1(x)+W_T(x) & \Omega_P(x) \\ \Omega_P(x) & W_2(x) - iW_Q(x)\end{pmatrix},$$

where $W_Q = \frac{\Omega_Q(x)^2}{\gamma_3}$. The second element of the approach is the resetting operation at each jump, $\exp\left(i\frac{mv_{rec}}{\hbar}u\mathbf{x}\right)\langle 3|\Psi(t)\rangle \longrightarrow \langle 1|\Psi(t)\rangle$, where $u \in [-1,1]$ is chosen with the probability density $\frac{3}{8}(1+u^2)$, all other amplitudes are set to zero, and the wave function is normalized. Because of Eq. (2), this can also be done in the two-level approximation, $-i\sqrt{W_Q(\mathbf{x})}\exp\left(i\frac{mv_{rec}}{\hbar}u\mathbf{x}\right)\langle 2|\Psi(t)\rangle \longrightarrow \langle 1|\Psi(t)\rangle$, then the second level is set to zero and the wave function is normalized.

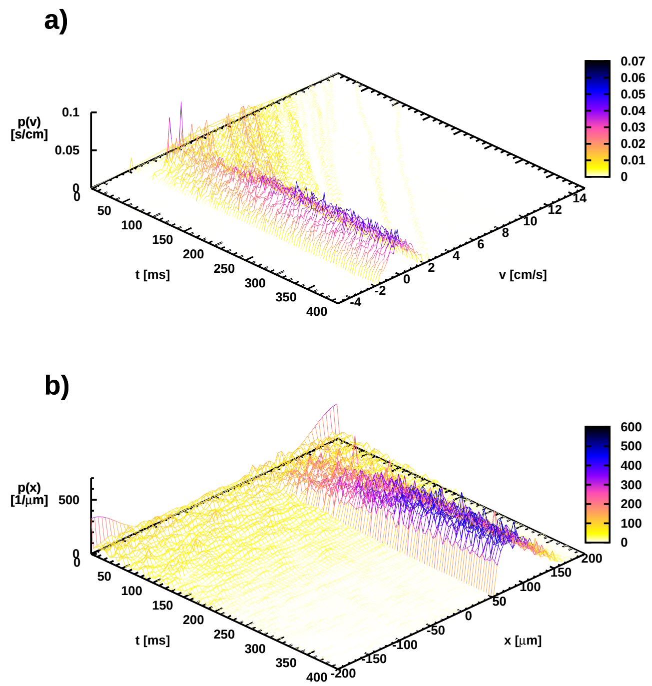

FIG. 3: Evolution of the probability densities versus time (a) in velocity space, (b) in coordinate space; $v_{rec} = 0$, the other parameters can be found in the caption of Fig. 2.

We start by looking at the case with negligible recoil velocity, i.e. with $v_{rec} = 0$. We calculate the trapping probability in coordinate space, $P_{T,x} = \int_{x_{min}}^{x_{max}} dx\,\langle x|\rho_{11}|x\rangle$, and in velocity space, $P_{T,v} = \int_{-v_T}^{v_T} dv\,\langle v|\rho_{11}|v\rangle$. The results are shown in Fig. 2b/c (thick green dotted line) averaging over $N = 200$ trajectories; a numerical error defined by the difference of the result between averaging over N and $N/2$ trajectories is also plotted in Fig. 2b/c. The parameters used for the atom diode with $v_{rec} = 0$ result in a range for perfect "diodic" behavior $-11\,$cm/s $< v < 11\,$cm/s (defined as in [3]). Fig. 3 shows the evolution of the probability density in velocity space, $p(v) = \langle v|(\rho_{11}+\rho_{22})|v\rangle$, and in coordinate space, $p(x) = \langle x|(\rho_{11}+\rho_{22})|x\rangle$. The final probability densities can be seen in more detail in Fig. 4 (thick, dashed, green line). In this figure we may verify the occurrence of cooling and phase space compression, namely, a narrower distribution both in coordinate and velocity space. The final trapping probabilities are

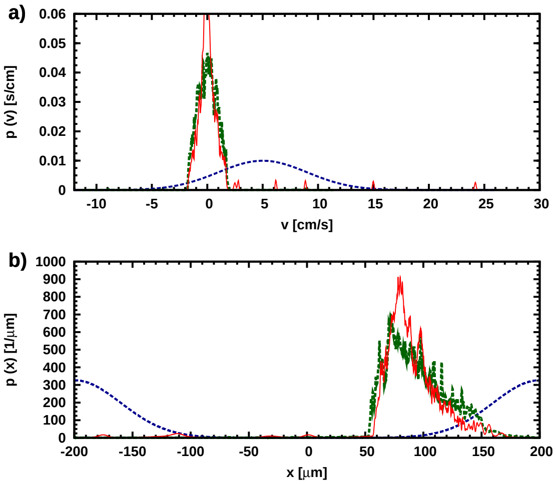

FIG. 4: Initial probability density (dotted blue line); final probability density at $t = 400\,\mathrm{ms}$ for $v_{rec} = 0$ (thick, green, dotted line), $v_{rec} = 3.5\,\mathrm{cm/s}$ (solid, red line); other parameters see Fig. 2; (a) in velocity space; (b) in coordinate space.

$P_{T,x} = 0.9838 \pm 0.0009$ and $P_{T,v} = 0.9809 \pm 0.0040$ (with the errors calculated as before).

Let us now examine the case with recoil velocity $v_{vec} = 3.5\,\mathrm{cm/s} > v_T$. Because the average value of the recoil velocities is still zero, the cooling method will still work except for a small fraction of atoms which may acquire by successive random recoils velocities above the break-up threshold of the diode. (The parameters used for the atom diode with $v_{rec} = 3.5\,\mathrm{cm/s}$ result in a working range $-17.5\,\mathrm{cm/s} < v < 22\,\mathrm{cm/s}$.) The trapping probability versus time shown in Fig. 2b (solid, red line) shows anyway a high final trapping probability. The final probability densities are shown in Fig. 4 (solid, red line). The main peak is comparable with the main peak without recoil, i.e. the velocity width of the main peak is smaller than the recoil velocity. We have finally $P_{T,x} = 0.9544 \pm 0.0003$ and $P_{T,v} = 0.9654 \pm 0.0017$.

In summary, we have proposed and numerically demonstrated a method to cool atoms on a ring, even below recoil velocity, after repeated passages across an atom diode combined with a ground state trap.

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* Email address: a.ruschhaupt@tu-bs.de

† Electronic address: jg.muga@ehu.es

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